TRANSVERSAL LOAD DISTRIBUTION ON BRIDGE DECKS

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SUMMARY

An orthotropic rectangular plate is analysed. The plate has been considered simply supported in two opposite edges and general boundary conditions along the remainder edges. Matrix formulation, very convenient for programming in digital computer, is used through the text. This technique is applied to an actual bridge deck and the results are compared with those obtained by means the Guyon-Massonet-Rowe method.

An alternative analysis using the folded plate structure theory is considered. This method represents a more sophisticated analysis, but appears to be also more approximate and suitable than the previous one in order to mathematically model actual bridges deck structures.
Comparative numerical results are presented.

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1.- ORTHOTROPIC PLATE THEORY

1.1. Introduction.

The analysis of some types of the bridges deck structures is usually divided in two major steps: 1) Longitudinal stress analysis (beam analysis) 2) Transversal load distribution.

This paper is mainly concerned to the last step. Some conventional analysis replaces during this study the actual bridge deck by an orthotropic plate. These ideas were first used, by Guyon, and successives improvements in the mathematical model were introducted by Massonet and Rowe among others. In reference (1) a complete analysis is presented and tables and charts for pratical applications are also included. Unfortunately, the method presented there, that will be call G.M.R. method, has several limitations pointed out in references (2) and (3).

In this paper an unified and consistent theory of the orthotropic plate, suitable for computer analysis is presented. The theory includes all the ranges of variation of the torsional parameter α, and it is given with some detail in ap 4.

1.2. Theory

The orthotropic plate governing equation is

$$k_{ij}w,_{iijj} = Z(x_1,x_2) \quad (i,j = 1,2)$$

and the stress-resultants are given by the following formulas:

$$m_{11} = -(k_{11}w,_{11} + k_1w,_{22}) \qquad m_{22} = -(k_{22}w,_{22} + k_2w,_{11})$$

$$m_{12} = -d_1w,_{12} \qquad m_{21} = -d_2w,_{12}$$

$$q_1 = -(k_{11}w,_{111} + (k_1+d_2)w,_{122}) \qquad q_2 = -(k_{22}w,_{222} + (k_2+d_1)w,_{122})$$

$$r_1 = q_1 + m_{12},_2 \qquad r_2 = q_2 + m_{21},_1$$

r_1 and r_2 are the Kirchoff shears.

Einstein's convention is used and the notation and sign conventions in figures 1 and 2.
The constant k_{ij} can be obtained from the actual properties of the bridge deck by the formulas:

$$\begin{aligned} k_{11} &= Ei & K_{22} &= Ej \\ d_1 &= Gi_o & d_2 &= Gj_o \\ k_1 &= \nu_1 k_{11} & k_2 &= \nu_2 k_{22} \\ 2k_{12} &= d_1 + d_2 + k_1 + k_2 \end{aligned} \qquad (2)$$

were i and j are the unit flexural inertias and i_o and j_o are the unit — torsional inertias along sections direction 1 and 2 respectively.
It is convenient to remember that the equation (1), assuming free b.c. — along the edges $x_2 = 0$, $x_2 = l_2$, is auto-adjoint if and only if $k_1 = k_2$ — (Betti theorem).

In order to obtain this result, the auto-adjoint property is equivalent, according to E.A. Coddington and N. Levinson "Theory of Ordinary Differential Equations" Mc Graw Hill 1955 to the following condition:

$$\underline{M}\,\underline{B}^{-1}(a)\underline{M}^T = \underline{N}\,\underline{B}^{-1}(b)\,\underline{N}^T \qquad (3)$$

where in this case: $a = 0$, $b = l_2$

$$\underline{B}(x_2) = \begin{bmatrix} 0 & p_2 & 0 & p_o \\ -p_2 & 0 & -p_o & 0 \\ 0 & p_o & 0 & 0 \\ -p_o & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{M} = \begin{bmatrix} 0 & m & 0 & -p_o \\ m' & 0 & -p_o & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \underline{N} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & n & 0 & -p_o \\ n' & 0 & -p_o & 0 \end{bmatrix}$$

$$p_o = k_{22} \qquad m = n = (2k_{12} - k_1)\lambda_k^2 \qquad \lambda_k = \frac{k\pi}{l_1}$$

$$p_2 = -2k_{12}\lambda_k^2 \qquad m' = n' = k_2\lambda_k^2$$

k an integer number

It is easy to see the equation (3) leads to the more simple condition - $k_1 = k_2$.

One of the more important difficulties on the application of the orthotropic plate theory to the bridge design is the traslation of mechanical properties of the actual deck into the parameters of the orthotropic plate - (k_{ij}).

In order to fulfil the Betti conditions sometimes the following assumption is used:

$$k_1 = k_2 = \nu k_{22}$$

or in another occasions:

$$k_1 = k_2 = \nu Eh^3/12$$

where h is the upper slab thickness and ν is Poisson ratio.

1.3. Example.

A computer program has been written according to the orthotropic plate - theory developped in the Appendix A.
The following example have been analysed, corresponding to the data (figure 4).

l_1 = 50 ft. = 18.29 m. $\qquad$ l_2 = 50 ft. = 15.24 m.
$\nu_1 = \nu_2$ = 0.15 $\qquad$ E = 2300000 t/m^2
i = 2057.2 in^4/in = 0.033711468 m^4/m
j = 422.0 " = 0.006915341 "
i_o = 189.0 " = 0.003103710 "
j_o = 52.0 " = 0.000855405 "

The orthotropic plate parameters corresponding to the equations (2) and the aditional condition

$$k_1 = k_2 = \nu k_{22}$$

Two loading conditions have been studied 1) Influence lines (Excentricity coefficientes). 2) Abnormal transportation loading.

1) Influence lines.

The acting "knife loading" has been aproximated by 1 or 5 Fourier serie (S.F.) terms. The results (vertical deflections w and longitudinal bending moments m_{11}) are represented in tables I and II.

The results A correspond to the G.M.R. method assuming the interpolation - formula for the excentricity coefficient:

$$K_{\alpha} = K_{0} + K_{1}\sqrt{\alpha}$$

where K_0 and K_1 are coefficient corresponding to extreme cases $\alpha = 0$ and - $\alpha = 1$ and are given in charts (reference (1)).

The results B and C are obtained by applying the orthotropic plate theory given here and not further approximation have introduced, except the linear elasticity hypothesis. Results B correspond to 1 S.F. term and results C - to 5 S.F. terms.

The excentricity coefficients given in the tables have been obtained from - the following definition:

The excentricity coefficient of a result r, k_r, at a given section is the ratio between the result r obtained by means orthotropic plate theory and the result r assuming beam theory, corresponding to the same loading condi tion.

In the G.M.R. it is assumed the following additional simplification:

$$k_m = k_w$$

In some text books (5) and (6) it is recommended the practice: $k_m = 1.1\ k_w$ if the G.M.R. theory is applied.

In the figures 6 and 7 the results from the tables I and II are represen- ted.

b) Abnormal transportation loading.

The extraordinary vehicle represented in figure 8 has been considered.

The following results have been obtained.

$$\text{Section: } x_1 = 0.416\, l_1 = 7.609 \text{ m.}$$
$$x_2 = 0.075\, l_2 = 1.143 \text{ m.}$$

G.M.R. : $m_{11} = 1.1 \times 5.742\ P = 6.020\ P$ mt/ml.

Orthotropic plate : $m_{11} = \qquad = 5.974\ P$ "

(The factor 1.1 has been used in the G.M.R.)

$$\text{Section: } x_1 = 0{,}5\, l_1 = 9.145 \text{ m.}$$
$$x_2 = 0{,}5\, l_2 = 7.620 \text{ m.}$$

G.M.R. : $m_{22} = 1.148\ P$ mt/ml.
Orthotropic plate : $m_{22} = 1.167\ P$ "

The position of the vehicle correspond in this case to the figure 9.

1.5. Comparison between G.M.R. and the orthotropic plate.

G.M.R. Method presents the following features:

a) The mathematical formulation is not suitable for programming.

b) In the free boundary conditions the Poisson ratio is assumed to be zero.

c) The validity of the empirical rules (only first S.F. term, factor 1.1) should be reconsidered.

d) Only results have been produced (tables, charts and interpolation formulas) for the range $0 \ll \alpha \ll 1$.

Contrarily the orthotropic plate theory needs in order to be applied a digital computer, but not presents the above limitations.

From the numerical results of the example it is observed.

a) The influence lines for w and m_{11} are not identical and they can not been obtained one from another by a simple scale factor.

b) The convergence of w is greater than the convergence of m_{11}.

c) In general, in the applications, both method would lead to the same results.

2. FOLDED PLATE STRUCTURES

2.1. Introduction

The orthotropic plate theory represents a consistent mathematical model - but in its application to the actual bridge deck analysis after difficulties arises :

First, about the translation to the geometric and elastic properties of - the deck into the orthotropic values k_{ij}. Second, about the interpretation of the results obtained from the orthotropic plate theory, where not distintion have been made among the diferent parts of the transversal section of the bridge, like webs, upper slab etc. The last difficulty is to obtain the in-plane stress distribution, because the orthotropic plate theory consider only flexural and torsional stress-resultants; that means, for example, is not possible to know the value of the shear-stress existing at the joint section between a web and a slab.

2.2. Folded plate structures theory.

The folded plate structures approach avoid most of the above mentioned -- difficulties, but increasing the computation cost. Prismatic folded plate structures theory has been very well stablished, in the elastic range. In the references (7) and (9) the more important features of the theory are shown.

Usually, for sake the analysis convenience, some simplifications into this theory are introducted (long prismatic folded plates), for example references (10) and (4).

2.3. Example

The bridge deck shown in figure 4, has been analysed alternatively by means of the orthotropic plate (O.P.) and the long prismatic folded plate (L.P. - F.P.) theories.

The parameters used in the O.P. theory were:

$l_1 = 18.19$ m. $\qquad l_2 = 15.24$ m.

$\nu_1 = \nu_2 = 0.15 \qquad E = 2300000\ t/m^2$

$i = 0.033711468\ m^4/m$

$j = 0.000281250$ "

$i_o = 0.003103710$ "

$j_o = 0.000562500$ "

and the Betti condition was assumed: $k_1 = k_2 = \nu_1 k_{22}$

The parameters of the L.P.F.P. theory are straighforward obtained, assuming - the idealised transversal section of the figure 10.

Similary to the example 1.3. the same two loading conditions have been considered 1) Influence lines 2) Abnormal transportation loading.

1) Influence lines.

In the tables V, VI, VII the results obtained from the two approach are -- shown. These compared results have been deflections (w), longitudinal normal stress (S) and transversal bending moments (M). In the table VIII are represented the values of the shear results (T) existing at the joint section of the upper slabs and the vertical webs. These results can be obtained only - from the L.P.F.P. theory.

In the figures 11, 12, 13, and 14, the above results have been drawn.

2) Abnormal transportation loading.

Assuming the vehicle position given in the fig. 8 the following results are obtained:

Section $x_1 = 0.416\ l_1 = 7.609$ m.

$x_2 = 0.075\ l_2 = 0.762$ m.

O.P. $m_{11} = 4.471$ mt/ml. and the corresponding stress at the flange level is:

$S = 8.8427 \times 4.471 = 40\ t/m^2$

L.P.F.P. Directly form the analysis $S = 53\ t/m^2$

Considering now the vehicles position of the figure 9, the results are:

Section $x_1 = 0.5\ l_1 = 9.145$ m.

$x_2 = 0.5\ l_2 = 7.620$ m.

O.P. $m_{22} = M = 0.280$ mt/ml

L.P.F.P. $M = 0.240$ "

2.4. Comparative study between O.P. and L.P.F.P. methods.

From the results of the above example same discrepancies between results - obtained from the two methods are observed. An explanation for this difference can be the relatively small span/width of the deck, particulary important in the L.P.F.P. theory.

By another hand, the L.P.F.P. mathematical model describes more accurately the actual behavior of the bridge deck than the O.P. model. In this last - theory only mean values of the stress and deflection are obtained as results and it is not distinghished between different parts of the transversal section. At end, the last disadvantage of the O.P. theory is its inability to give any idea about in-plane stress distribution.

3. Conclusions.

Although a more exhaustive study is needed in order to obtain a more definitive conclusions, particulary in reference to the importance of the span/width ratio, number and distance between longitudinal beams, slab thickness, type of transversal section etc, the following provisional statements can be presented:

1) The orthotropic plate theory appears to be a more consistent mathematical model that the previous one Guyon-Massonet-Rowe.

2)Both methods, orthotropic plate and Guyon-Massonet-Rowe, present in its applications, the difficults to model correctly the most relevant properties of an actual bridge deck structure.

3) The folded plate structure represents a more advance theory in order to model this type of bridge structures than the two above mentioned theories. Particulary the in-plane stress are given from this approach and irregular types of transversal section can be handled without additional computational effort.

4) If the number of longitudinal beams is elevated and the distance between them is small the orthotropic plate theory may represent accurately enough the actual bridge deck behaviour. In other case this statement can not be valid.

TABLE I

DEFLECTION EXCENTRICITY COEFICIENTS k_w SECTION $x_1 = 0.5\ l_1$

CONSIDERED SECTION	LOADED SECTION									Type results
	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000	
0	4.80	3.46	2.01	1.02	0.37	-0.04	-0.30	-0.52	-0.69	A
	5.35	3.61	2.17	1.12	0.41	-0.05	-0.35	-0.57	-0.77	B
	5.30	3.59	2.18	1.12	0.41	-0.05	-0.35	-0.57	-0.77	C
0.125	3.45	2.65	1.92	1.26	0.71	0.31	-0.02	-0.26	-0.51	A
	3.61	2.81	2.00	1.28	0.71	0.28	-0.05	-0.32	-0.57	B
	3.59	2.79	1.99	1.28	0.71	0.28	-0.05	-0.32	-0.57	C
0.250	2.01	1.92	1.77	1.43	1.02	0.66	0.30	-0.02	-0.30	A
	2.17	2.00	1.79	1.43	1.01	0.62	0.27	-0.05	-0.35	B
	2.18	1.99	1.78	1.42	1.01	0.62	0.27	-0.05	-0.35	C
0.375	1.02	1.26	1.44	1.48	1.32	1.01	0.66	0.31	-0.04	A
	1.12	1.28	1.43	1.47	1.28	0.96	0.58	0.28	-0.05	B
	1.12	1.28	1.42	1.45	1.27	0.96	0.62	0.28	-0.05	C
0.500	0.37	0.71	1.02	1.32	1.45	1.32	1.02	0.71	0.37	A
	0.41	0.71	1.01	1.28	1.40	1.28	1.01	0.71	0.41	B
	0.41	0.71	1.01	1.27	1.39	1.27	1.01	0.71	0.41	C

TABLE II

BENDING MOMENT EXCENTRICITY COEFICIENTS k_m SECTION $x_1 = 0.5\ l_1$

CONSIDERED SECTION	LOADED SECTION									Type results
	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000	
0	4.80	3.46	2.01	1.02	0.37	-0.04	-0.30	-0.52	-0.69	A
	5.48	3.69	2.23	1.15	0.42	-0.05	-0.36	-0.58	-0.78	B
	5.10	3.57	2.24	1.17	0.42	-0.05	-0.36	-0.58	-0.78	C
0.125	3.45	2.65	1.92	1.26	0.71	0.31	-0.02	-0.26	-0.51	A
	3.61	2.91	2.07	1.32	0.72	0.28	-0.05	-0.33	-0.59	B
	3.50	2.80	2.00	1.31	0.73	0.28	-0.05	-0.33	-0.59	C
0.250	2.01	1.92	1.77	1.43	1.02	0.66	0.30	-0.02	-0.30	A
	2.13	2.03	1.90	1.50	1.05	0.63	0.26	-0.07	-0.38	B
	2.14	1.97	1.80	1.44	1.04	0.64	0.27	-0.07	-0.38	C
0.375	1.02	1.26	1.44	1.48	1.32	1.01	0.66	0.31	-0.04	A
	1.05	1.27	1.49	1.59	1.35	1.00	0.62	0.25	-0.10	B
	1.07	1.26	1.42	1.49	1.29	0.99	0.63	0.26	-0.10	C
0.500	0.37	0.71	1.02	1.32	1.45	1.32	1.02	0.71	0.37	A
	0.34	0.69	1.03	1.35	1.53	1.35	1.03	0.69	0.34	B
	0.35	0.69	1.02	1.29	1.43	1.29	1.02	0.69	0.35	C

TABLE V

VERTICAL DEFLECTIONS w (mm)

LOAD POSITION		CONSIDERED SECTION												
		1	2	6	10	14	18	22	26	30	34	38	42	
	1	11.15	8.22	3.60	1.05	0.01	-0.26	-0.23	-0.13	-0.06	-0.01	-0.01	0.02	ORTHOTROPIC PLATE
	2	8.22	6.80	3.81	1.60	0.43	-0.03	-0.13	-0.11	-0.06	-0.02	-0.00	0.01	
	2-6	5.65	5.27	3.97	2.17	0.89	0.22	-0.03	-0.08	-0.06	-0.04	-0.01	-0.00	
	6	3.60	3.81	3.91	2.76	1.41	0.53	0.11	-0.04	-0.06	-0.05	-0.02	-0.01	
	6-10	2.08	2.56	3.44	3.25	2.00	0.92	0.30	0.03	-0.05	-0.06	-0.04	-0.03	
	10	1.05	1.59	2.76	3.44	2.62	1.41	0.56	0.13	-0.03	-0.06	-0.06	-0.06	
	10-14	0.39	0.90	2.04	3.17	3.14	1.98	0.93	0.31	0.03	-0.06	-0.08	-0.09	
	14	0.01	0.43	1.41	2.62	3.67	2.59	1.40	0.57	0.13	-0.04	-0.11	-0.13	
	14.18	-0.19	0.13	0.91	1.99	3.12	3.11	1.97	0.93	0.30	0.01	-0.13	-0.18	
	18	-0.26	-0.03	0.53	1.41	2.59	3.43	2.58	1.40	0.56	0.11	-0.13	-0.23	
	18.22	-0.26	-0.11	0.27	0.93	1.97	3.10	3.10	1.97	0.93	0.27	-0.11	-0.26	
	1	13.40	9.50	3.59	0.78	-0.07	-0.16	-0.10	-0.11	-0.19	-0.34	-0.51	-0.60	FOLDED PLATE
	2	9.50	7.52	3.84	1.49	0.43	0.08	-0.02	-0.07	-0.16	-0.29	-0.44	-0.51	
	2 -6	6.15	5.58	4.06	2.19	0.94	0.32	0.07	-0.04	-0.13	-0.24	-0.36	-0.43	
	6	3.59	3.84	4.01	2.84	1.47	0.60	0.18	-0.00	-0.10	-0.19	-0.29	-0.34	
	6-10	1.85	2.48	3.55	3.37	2.06	0.95	0.33	0.06	-0.06	-0.14	-0.22	-0.26	
	10	0.78	1.49	2.84	3.56	2.66	1.39	0.56	0.15	-0.01	-0.01	-0.16	-0.19	
	10-14	0.20	0.84	2.11	3.26	3.16	1.94	0.88	0.30	0.05	-0.05	-0.11	-0.14	
	14	-0.07	0.43	1.47	2.66	3.37	2.52	1.32	0.53	0.15	0.00	-0.07	-0.11	
	14-18	-0.16	0.20	0.97	1.99	3.09	3.05	1.88	0.87	0.31	0.07	-0.04	-0.09	
	18	-0.16	0.08	0.60	1.39	2.52	3.27	2.48	1.32	0.56	0.18	-0.02	-0.10	
	18-22	-0.13	-0.02	-0.85	0.91	1.90	3.02	3.02	1.90	0.91	0.35	-0.02	-0.13	

TABLE VI

LONGITUDINAL NORMAL STRESSES S (t/m^2)

	LOAD POSITION	CONSIDERED SECTION											
		1	2	6	10	14	18	22	26	30	34	38	42
ORTHOTROPIC PLATE	1	211	160	73	21	−0	−5	−5	−3	−1	−0	−0	−1
	2	160	132	76	32	8	−1	−3	−2	−1	−0	−0	−1
	2–6	113	103	77	44	18	4	−1	−2	−1	−1	−1	−1
	6	73	76	76	55	28	11	2	−1	−1	−1	−1	−1
	6–10	42	52	67	63	41	18	6	0	−1	−1	−1	−1
	10	21	32	55	67	52	28	11	3	−0	−1	−2	−2
	10–14	8	18	41	61	61	40	19	6	0	−1	−2	−3
	14	0	9	29	52	65	51	28	11	3	−1	−3	−4
	14–18	−4	3	18	40	60	60	40	19	6	0	−3	−5
	18	−5	−1	11	28	51	64	51	28	11	2	−2	−5
	18–22	−5	−1	6	19	40	60	60	40	19	6	−1	−5
FOLDED PLATE	1	127	121	84	47	22	8	0	−5	−9	−15	−24	−30
	2	112	106	77	48	26	12	4	−2	−7	−13	−20	−26
	2–6	96	91	71	50	31	16	7	1	−5	−10	−17	−22
	6	80	76	64	51	35	21	11	3	−2	−7	−13	−18
	6–10	65	63	58	50	39	26	14	6	0	−5	−10	−14
	10	52	50	50	48	42	30	19	10	3	−2	−7	−10
	10–14	40	39	43	46	43	33	24	14	6	−1	−4	−6
	14	30	30	35	41	43	38	29	18	10	4	−1	−3
	14–18	22	22	28	36	41	41	34	23	14	7	3	1
	18	15	16	21	30	38	41	33	29	19	11	6	5
	18–22	10	10	16	24	34	40	40	34	24	16	10	10

TABLE VII

TRANSVERSAL BENDING MOMENTS M (mt/ml)

		PLATE 1		PLATE 2		PLATE 3		PLATE 4		PLATE 5		PLATE 6		PLATE 7		PLATE 8		PLATE 9		PLATE 10		PLATE 11	
		1	2_1	2_2	6_1	6_2	10_1	10_2	14_1	14_2	18_1	18_2	22_1	22_2	26_1	26_2	30_1	30_2	34_1	34_2	38_1	38_2	42
LOAD POSITION	1	0.0	-0.39	-0.39	-0.60	-0.60	-0.42	-0.42	-0.21	-0.21	-0.08	-0.08	-0.01	-0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.00	0.00	0.0
	2	0.0	0.26	-0.26	-0.24	-0.24	-0.30	-0.30	-0.20	-0.20	-0.09	-0.09	-0.03	-0.03	-0.03	-0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.0
	2-6	0.0	0.14	0.14	-0.14	0.14	-0.16	-0.16	-0.18	-0.18	-0.11	-0.11	-0.05	-0.05	-0.02	-0.02	-0.00	-0.00	0.00	0.00	0.00	0.00	0.0
	6	0.0	0.06	0.06	0.58	0.58	0.03	0.03	-0.14	-0.14	-0.13	-0.13	-0.08	-0.05	-0.03	-0.03	-0.01	-0.01	0.00	0.00	0.00	0.00	0.0
	6-10	0.0	-0.00	-0.00	0.30	0.30	0.29	0.29	-0.06	-0.06	-0.13	-0.13	-0.10	-0.10	-0.05	-0.05	-0.02	-0.02	0.00	0.00	0.00	0.00	0.0
	10	0.0	-0.03	-0.03	0.11	0.11	0.66	0.66	-0.08	-0.08	-0.11	-0.11	-0.12	-0.12	-0.07	-0.07	-0.03	-0.03	-0.01	0.01	0.00	0.00	0.0
	10-14	0.0	-0.04	-0.04	-0.01	-0.01	-0.03	0.33	0.32	0.32	-0.05	-0.05	-0.13	-0.13	-0.10	-0.10	-0.05	-0.05	-0.02	0.02	0.00	0.00	0.0
	14	0.0	-0.04	-0.04	-0.07	-0.07	0.10	0.10	0.66	0.66	0.09	0.09	-0.11	-0.11	-0.12	-0.12	-0.07	-0.07	-0.03	-0.03	-0.00	-0.00	0.0
	14-18	0.0	-0.04	-0.04	-0.09	-0.09	-0.03	-0.03	0.32	0.32	0.32	0.32	-0.05	-0.05	-0.12	-0.12	-0.09	-0.09	-0.04	-0.04	-0.01	-0.01	0.0
	18	0.0	-0.03	-0.03	-0.09	-0.09	-0.10	-0.10	0.09	0.09	0.66	0.66	0.09	0.09	-0.11	-0.11	-0.11	-0.11	-0.06	-0.06	-0.01	-0.01	0.0
	18-22	0.0	-0.02	-0.02	-0.08	-0.08	-0.12	-0.12	-0.04	-0.04	0.32	0.32	0.32	0.32	-0.04	-0.04	-0.12	-0.12	-0.08	-0.08	-0.02	-0.02	0.0
	1	0.0	-0.84	-0.79	-0.96	-0.96	-0.53	-0.52	-0.18	-0.18	-0.02	-0.02	0.02	+0.02	0.02	+0.02	0.01	+0.01	0.00	+0.00	-0.00	-0.00	0.0
	2	0.0	0.00	+0.00	-0.46	-0.45	-0.38	-0.37	-0.18	-0.18	-0.06	-0.05	-0.00	0.00	0.01	+0.01	0.01	+0.01	0.00	+0.00	-0.00	-0.00	0.0
	2-6	0.0	0.00	+0.20	0.24	+0.04	-0.21	-0.21	-0.19	-0.18	-0.09	-0.09	-0.03	-0.03	-0.00	0.00	0.00	+0.00	0.00	+0.00	-0.00	+0.00	0.0
	6	0.0	-0.00	+0.00	0.56	+0.56	-0.01	-0.01	-0.17	-0.17	-0.13	-0.13	-0.06	-0.06	-0.01	-0.01	0.00	+0.00	0.00	+0.00	-0.00	-0.00	0.0
	6-10	0.0	-0.00	+0.00	0.30	+0.50	0.47	+0.27	-0.11	-0.10	-0.15	-0.15	-0.09	-0.09	-0.03	-0.03	-0.00	0.00	0.00	+0.00	-0.00	-0.00	0.0
	10	0.0	0.00	0.00	0.10	+0.10	0.63	+0.64	0.03	+0.03	-0.15	-0.15	-0.12	-0.12	-0.06	-0.06	-0.01	-0.01	-0.00	-0.00	-0.00	+0.00	0.0
	10-14	0.0	0.00	0.00	-0.01	-0.01	0.30	+0.50	0.47	+0.27	-0.10	-0.10	-0.15	-0.14	-0.09	-0.09	-0.03	-0.03	-0.00	-0.00	-0.00	+0.00	0.0
	14	0.0	0.00	0.00	-0.07	-0.07	0.06	+0.06	0.62	+0.62	0.03	+0.03	-0.15	-0.14	-0.12	-0.12	-0.05	-0.05	-0.01	-0.01	-0.00	+0.00	0.0
	14-18	0.0	0.00	0.00	-0.08	-0.09	-0.07	-0.07	0.27	+0.47	0.47	+0.27	-0.09	-0.09	-0.14	-0.14	-0.08	-0.08	-0.03	-0.03	-0.00	+0.00	0.0
	18	0.0	0.00	0.00	-0.08	-0.08	-0.12	-0.13	0.03	+0.03	0.62	+0.62	0.03	+0.03	-0.14	-0.14	-0.11	-0.11	-0.04	-0.04	-0.00	0.00	0.0
	18-22	0.0	0.00	-0.00	-0.06	-0.06	-0.13	-0.13	-0.09	-0.09	0.27	0.47	0.47	0.27	-0.09	-0.09	-0.13	-0.13	-0.06	-0.06	-0.00	0.00	0.0

TABLE VIII

SHEAR-RESULTS T (t) SECTION $x_1 = 0.\ l_1$

LOAD POSITION	PLATE 1		PLATE 2		PLATE 3		PLATE 4		PLATE 5		PLATE 6		PLATE 7		PLATE 8		PLATE 9		PLATE 10		PLATE 11	
	1	2_1	2_2	6_1	6_2	10_1	10_2	14_1	14_2	18_1	18_2	22_1	22_2	26_1	26_2	30_1	30_2	34_1	34_2	38_1	38_2	42
1	0.0	3.33	9.18	-4.17	5.68	-3.07	2.19	-0.77	-0.44	1.11	-1.85	2.09	-2.37	2.34	-2.38	2.14	-2.09	1.63	-1.52	0.76	-0.53	0.0
2	0.0	2.33	6.40	-1.87	4.83	-2.13	2.33	-0.79	0.23	0.58	-1.09	1.45	-1.72	1.80	-1.89	1.74	-1.73	1.36	-1.29	0.65	-0.45	0.0
2-6	0.0	2.25	3.65	0.37	4.06	-1.20	2.50	-0.81	0.92	0.04	-0.31	0.81	-1.07	1.25	-1.39	1.33	-1.37	1.10	-1.06	0.54	-0.54	0.0
6	0.0	1.68	1.35	2.11	3.01	-0.04	2.63	-0.74	1.56	-0.42	0.44	0.20	-0.41	0.70	-0.88	0.92	-1.01	0.84	-0.83	0.43	-0.30	0.0
6-10	0.0	1.29	-0.05	2.84	1.32	1.54	2.65	-0.46	2.09	-0.75	1.16	-0.36	0.24	0.17	-0.37	0.51	-0.65	0.57	-0.61	0.32	-0.23	0.0
10	0.0	0.99	-0.78	2.92	-0.29	2.91	2.17	0.26	2.45	-0.86	1.80	-0.81	0.90	-0.34	0.15	0.10	-0.27	0.30	-0.38	0.20	-0.16	0.0
10-14	0.0	0.75	-1.12	2.77	-1.21	3.37	0.87	1.58	2.62	-0.68	2.31	-1.11	1.53	-0.80	0.70	-0.31	0.11	0.02	-0.14	0.08	-0.10	0.0
14	0.0	0.56	-1.17	2.44	-1.58	3.25	-0.50	2.81	2.22	0.01	2.65	-1.18	2.10	-1.17	1.26	-0.72	0.54	-0.28	0.10	-0.05	-0.03	0.0
14-18	0.0	0.40	-1.06	2.02	-1.65	2.95	-1.28	3.20	0.97	1.34	2.80	-0.95	2.55	-1.40	1.82	-1.10	1.01	-0.60	0.40	-0.21	0.02	0.0
18	0.0	0.23	-0.85	1.56	-1.51	2.53	-1.57	3.06	-0.38	2.58	2.39	-0.21	2.86	-1.41	2.34	-1.41	1.51	-0.93	0.73	-0.41	0.09	0.0
18-22	0.0	0.18	-0.63	1.13	-1.26	2.04	-1.58	2.77	-1.14	2.99	1.13	1.13	2.99	-1.14	2.77	-1.58	2.04	-1.26	1.13	-0.63	0.18	0.0

FOLDED PLATE

Appendix A

General Theory of the Orthotropic Plate.

Governing equation

$$k_{ij}\, w_{,iijj} = Z(x_1, x_2) \qquad (i,j = 1,2) \qquad (I)$$

Stress-resultants

$m_{11} = -(k_{11}w_{,11} + k_1 w_{,22})$ $\qquad$ $m_{22} = -(k_{22}w_{,22} + k_2 w_{,11})$

$m_{22} = -\,d_1 w_{,12}$ $\qquad$ $m_{21} = -\,d_2 w_{,12}$

$q_1 = -(k_{11}w_{,111} + (k_1+d_2)w_{,122})$ $\qquad$ $q_2 = -(k_{22}w_{,222} + (k_2+d_1)w_{,122})$

$r_1 = q_1 + m_{12,2}$ $\qquad$ $r_2 = q_2 + m_{21,1}$

Complementary solution.

By definition the complementary solution is the solution of the homogenlous equation (1) i.e.

$$k_{ij}\, w_{c,iijj} = 0 \qquad (II)$$

Assuming simply support along $x_1 = 0$ and $x_1 = l_1$ the complementary solution can be written as:

$$w_c = \sum_{n=1}^{\infty} w_n^*(x_2)\sin\lambda_n x_1 \qquad \text{where } \lambda_n = \frac{n\pi}{l_1}$$

Substituying into (II) and considering the n-th term it is obtained

$$k_{11}\lambda_n^4 w_n^*(x_2) + 2k_{12}\lambda_n^2 w^*_{n,22}(x_2) + k_{22}w^*_{n,2222}(x_2) = 0 \qquad (III)$$

or for convenience the subscript n can be dropped in all the following - formulas and the equation (III) becomes:

$$k_{11}\lambda^4 w^* - 2k_{12}\lambda^2 w^*_{,22} + k_{22}\, w^*_{,2222} = 0 \qquad (IV)$$

The algebraic roots t_j $(j = 1,2,3,4)$ of the characteristic equation (IV) are

$$t_j = \pm\sqrt{\frac{k_{12} \pm \sqrt{\Delta}}{k_{22}}} \qquad \text{where } \Delta = k_{12}^2 - k_{11}k_{22} \qquad (V)$$

The following posible three cases are considered.

Case I. $\Delta<0$ Torsionally soft and flexurally stiff orthotropic plates. (Typical case in bridge analysis).

Case II. $\Delta=0$ Isotropic plate.

Case III. $\Delta>0$ Torsionally stiff and flexurally soft orthotropic plate.

Introducing the definition of the flexural parameter $\theta = \sqrt[4]{\dfrac{k_{11}}{k_{12}}}$

The equation (V) becomes:

Case I. $t_j = \pm \lambda\theta e^{\pm\frac{\alpha}{2}i} = \pm (r \pm si)$ $\qquad i = \sqrt{-1}$

where $\alpha = \text{arc cos}\,\dfrac{k_{12}}{k_{11}\,k_{22}}$ $\qquad r = \lambda\theta\cos\frac{\alpha}{2} \quad s = \lambda\theta\sin\frac{\alpha}{2}$

Case II. $t_j = \pm\lambda\theta$ (multiple roots)

Case III. $t_j = \pm\lambda\theta e^{\pm\frac{\alpha}{2}} = \pm (r \pm s)$

where $\alpha = \text{arg Ch}\,\dfrac{k_{12}}{k_{11}\,k_{22}}$ $\qquad r = \lambda\theta\,\text{Ch}\frac{\alpha}{2} \quad s = \lambda\theta\,\text{Sh}\,\frac{\alpha}{2}$

The complementary solution is obtained from the following formulas:

$$\underline{R}_c\,(x_2) = \underline{G}\left[\underline{B}\,\underline{P}\,(x_2),\ \underline{C}\,\underline{P}(l_2-x_2)\right]\underline{A}_{1234}$$

where: $\underline{R}_c\,(x_2) = (w;w,_1;w,_2;m_{11};m_{22};m_{12};m_{21};q_1;q_2;r_1;r_2)^t$

Each term of $\underline{R}_c\,(x_2)$ is a function of x_2 and varies along the direction x_1 as $\sin\lambda x_1$ except $w,_1;m_{12};m_{21};q_1;r_1$ wich vary as $\cos\lambda x_1$.

$$\underline{G} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ k_{11}\lambda^2 & 0 & -k_1 & 0 \\ k_2\lambda^2 & 0 & -k_{22} & 0 \\ 0 & -d_{12}\lambda & 0 & 0 \\ 0 & -d_{21}\lambda & 0 & 0 \\ k_{11}\lambda^3 & 0 & -(k_1+d_{21})\lambda & 0 \\ 0 & (k_2+d_{12})\lambda & 0 & -k_{22} \\ k_{11}\lambda^3 & 0 & -(2k_{12}-k_2)\lambda & 0 \\ 0 & (2k_{12}-k_1)\lambda^2 & 0 & -k_{22} \end{bmatrix}$$

$$\underline{A}_{1234} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix} \qquad A_i \ (i = 1,2,3,4) \text{ are arbitrary constants.}$$

The matrices $\underline{B}$ and $\underline{P}\ (x_2)$ depend on the orthotropic plate case considered:

Case I.

$$\underline{B} = \begin{bmatrix} 1 & 0 \\ -r & s \\ r^2-s^2 & -2rs \\ -r^3 + 3rs^2 & -s^3+3sr^2 \end{bmatrix} \qquad \underline{P}\ (x_2) = e^{-rx_2} \begin{bmatrix} \cos sx_2 & \sin sx_2 \\ -\sin sx_2 & \cos sx_2 \end{bmatrix}$$

Case II.

$$\underline{P} = \begin{bmatrix} 1 & 0 \\ -r & 1 \\ r^2 & -2r \\ -r^3 & 3r^2 \end{bmatrix} \qquad \underline{P}(x_2) = e^{-rx_2} \begin{bmatrix} 1 & x_2 \\ 0 & 1 \end{bmatrix}$$

Case III.

$$\underline{B} = \begin{bmatrix} 1 & 0 \\ -r & s \\ r^2+s^2 & -2rs \\ -r^3-3rs^2 & s^3+3sr^2 \end{bmatrix} \qquad \underline{P}\ (x_2) = e^{-rx_2} \begin{bmatrix} \mathrm{Ch}\ sx_2 & \mathrm{Sh}\ sx_2 \\ \mathrm{Sh}\ sx_2 & \mathrm{Ch}\ sx_2 \end{bmatrix}$$

and for all the cases $\underline{C} = \underline{\delta}\ \underline{B}$

where $\underline{\delta} = \left\{\delta_{ij}\right\}$ $(i,j = 1,2,3,4)$ and $\delta_{ij} = 0 \quad \delta_{ii} = (-1)^{i+1}$

Particular solution.

By definition, it is the solution of equation (I) not satisfiying all the boundary conditions. It is assumed here the orthotropic plate simply supportted at $x_1 = 0$ and $x_1 = l_1$ and infinite width.

Two loading conditions are considered.

a) Knife load along $x_2 = \alpha_2$ i.e.

$Z(x_1,x_2) = Z(x_2) \cdot \delta(x_2-\alpha_2)$

In this case it can be shown that the particular solution is:

$$\underline{R}_o(x_2) = \begin{cases} \underline{G}\,\underline{C}\,\underline{P}(\alpha_2-x_2)\,\underline{A}^o_{12} & \text{if } x_2 \ll \alpha_2 \\ \underline{G}\,\underline{B}\,\underline{P}(x_2-\alpha_2)\,\underline{A}^o_{34} & \text{if } x_2 > \alpha_2 \end{cases}$$

where $\underline{R}_o(x_2)$ is a similar matrix as $\underline{R}_c(x_2)$.

The matrices $\underline{G}$, $\underline{B}$, $\underline{C}$, $\underline{P}(x_2)$ have been already defined

$$\underline{A}^o_{12} = \underline{A}^o_{34} = \begin{bmatrix} 1/r \\ 1/s \end{bmatrix} . H \qquad \text{Cases I y III.}$$

$$\underline{A}^o_{12} = \underline{A}^o_{34} = \begin{bmatrix} 1/r \\ 1 \end{bmatrix} . H \qquad \text{Case II.}$$

where $H = \dfrac{a_n}{4\lambda\sqrt{k_{11}k_{12}}}$ and $a_n = \dfrac{2}{l_1} \cdot \displaystyle\int_0^{l_1} Z(x)\sin\lambda x.dx$

in the n-th S.F. term of $Z(x_1)$.

The boundary conditions to be considered are:

$$\begin{array}{ll} |x_2| \to \infty & w \to 0 \\ x_2 \to \alpha_2 \pm 0 & w,_2 \to 0 \\ x_2 \to \alpha_2 \pm 0 & w,_{222} = \dfrac{a_n}{2k_{11}} \end{array}$$

b) Uniforme distributed load, i.e. $Z(x_1,x_2) = Z(x_1)$

The particular solution is:

$\underline{R}_o(x_2) = \underline{G}\,\underline{B}_o \dfrac{a_n}{k_{11}\lambda^4}$ where $\underline{B}_o = \underline{B}$ (assuming r=s=0 in every element of $\underline{B}$)

Final solution.

Is the solution of equation (I) plus the actual boundary conditions, and - its expresion is:

$$\underline{R}(x_2) = \underline{R}_o(x_2) + \underline{R}_c(x_2) \qquad (VI)$$

The matrix $\underline{A}_{1234}$ of $\underline{R}_c\ (x_2)$ is obtained from the boundary conditions, - along $x_2=0$ and $x_2=l_2$. Two cases of the boundary conditions are considered:

a) Homogeneous boundary conditions.

The general matrix form of this conditions is given for each edge i (i=1,2: i.e. $x_2=0$ or $x_2=l_2$ respectively).

$$\underline{k}_{di}\begin{bmatrix}-w,_2\\ w\end{bmatrix} + \underline{k}_{pi}\begin{bmatrix}m_{22}\\ r_2\end{bmatrix} = \underline{0} \qquad \text{(VII)}$$

The diagonal matrix $\underline{k}_{di}$ and $\underline{k}_{pi}$ satisfy the condition

$\underline{k}_{di} + \underline{k}_{pi} = \underline{I}_2$ (unit matrix)

The unknowns $\underline{A}_{1234}$ are obtained from the following equation:

$$\begin{bmatrix} \underline{k}_{d1}\begin{bmatrix}\underline{\alpha}_{13}\\ \\ \underline{\alpha}_{11}\end{bmatrix} + \underline{k}_{p1}\begin{bmatrix}\underline{\alpha}_{15}\\ \\ \underline{\alpha}_{1\ 11}\end{bmatrix} \\ \underline{k}_{d2}\begin{bmatrix}\underline{\alpha}_{23}\\ \\ \underline{\alpha}_{21}\end{bmatrix} + \underline{k}_{p2}\begin{bmatrix}\underline{\alpha}_{25}\\ \\ \underline{\alpha}_{2\ 11}\end{bmatrix}\end{bmatrix}\underline{A}_{1234} = -\begin{bmatrix} \underline{k}_{d1}\begin{bmatrix}\beta_{13}\\ \\ \beta_{11}\end{bmatrix} + \underline{k}_{p1}\begin{bmatrix}\beta_{15}\\ \\ \beta_{1\ 11}\end{bmatrix} \\ \underline{k}_{d2}\begin{bmatrix}\beta_{23}\\ \\ \beta_{21}\end{bmatrix} + \underline{k}_{p2}\begin{bmatrix}\beta_{25}\\ \\ \beta_{2\ 11}\end{bmatrix}\end{bmatrix}$$

where: $\underline{\alpha}_{1i}$ is the i-row matrix of $\left[\underline{G}\ \underline{B},\ \underline{G}\ \underline{C}\ \underline{P}\ (l_2)\right]$

$\underline{\alpha}_{2i}$ " " " " " $\left[\underline{G}\ \underline{B}\ \underline{P}\ (l_2),\ \underline{G}\ \underline{C}\right]$

β_{1i} is the i-element of $\underline{G}\ \underline{C}\ \underline{P}\ (\alpha_2)\ \underline{A}^{o}_{12}\ \sigma\ \underline{G}\ \underline{B}_o\ \dfrac{a_n}{k_{11}\lambda^4}$

β_{2i} " " " " $\underline{G}\ \underline{B}\ \underline{P}\ (l_2-\alpha_2)\ \underline{A}^{o}_{34}\ \sigma\ \underline{G}\ \underline{B}_o\ \dfrac{a_n}{k_{11}\lambda^4}$

depending on the loading case considered.

Then, the final solution $\underline{R}\ (x_2)$ is obtained from (VI).

b) Edge beam.

The stiffnes matrix, in local axis, of the edge beam i (i=1,2) is:

$$\begin{bmatrix} m_{22} \\ \\ r_2 \end{bmatrix}_{beam} = \underline{R}_1 \begin{bmatrix} -w,_2 \\ \\ w \end{bmatrix}_{beam} \qquad \text{(VIII)} \qquad \text{where} \quad \underline{R}_i = \begin{bmatrix} -\lambda^2 GJ_i & 0 \\ \\ 0 & \lambda^4 EI_i \end{bmatrix}$$

EI_i, GJ_i are the flexural and torsional stiffness of the beam.

Using the standard axis transformation technique (rotation and traslation shown in figure 3) the equation (VIII) becomes:

$$\begin{bmatrix} m_{22} \\ \\ r_2 \end{bmatrix}_{plate} = \underline{T}_i \underline{R}_i \underline{T}_i^t \begin{bmatrix} -w,_2 \\ \\ w \end{bmatrix}_{plate} \qquad \text{(IX)}$$

where $\underline{T}_i = \begin{bmatrix} 1 & -a_i \cos\alpha_i + b_i \sin\alpha_i \\ 0 & \cos\alpha_i \end{bmatrix}$ is the transformation matrix.

The equation (IX) have the same mathematical structure as (VII) where: $\underline{k}_{pi} = \underline{I}_2$ and $\underline{k}_{\pi i} = \underline{T}_i \underline{R}_i \underline{T}_i^t$ and then, the solution steps will be similar to the previous case.

N O T A T I O N

x_1, x_2, z	Orthogonal cartesian coordinates
$Z(x_1, x_2)$	Vertical surface load on (x_1, x_2) per unit area
$v_{,i}$	Derivative of v respect x_i
$\underline{V}$	V variable is a matrix
w	vertical displacement of the plate middle surface
m_{ij}	Force acting on the face j in i direction. (bending or torsor moment) $(i, j = 1.2)$
q_i	shear force acting on the face i
r_i	Kirchoff shear
E	Young's modulus
ν_i	Poisson's ratio, in i direction
l_1, l_2	Lengths of the plate (figure 1) (span and width respectively)
$\delta(x_2 - \alpha)$	Dirac delta distribution, point α and x_2 variable
K_o K_1 K_α	Distribution coeficients of the G.M.R. method correspon ding to orthotropic plate where $\alpha = 0, 1, \alpha$ respectively
k_w	Deflection excentricity coeficient
k_m	Bending moment excintricity coeficient
S.F.	Fourier serie
G.M.R.	Guyon-Massonet-Rowe
L.P.F.P.	Long prismatic folded plate
O.P.	Orthotropic plate

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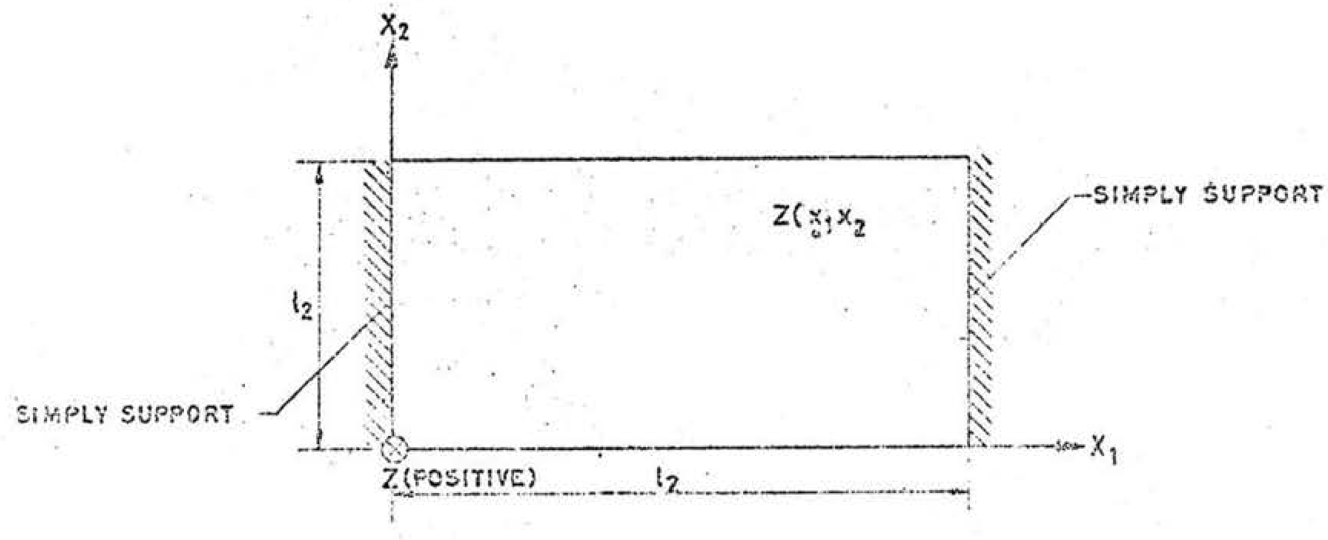


FIGURE 1. PLAN OF THE RECTANGULAR ORTHOTROPIC PLATE

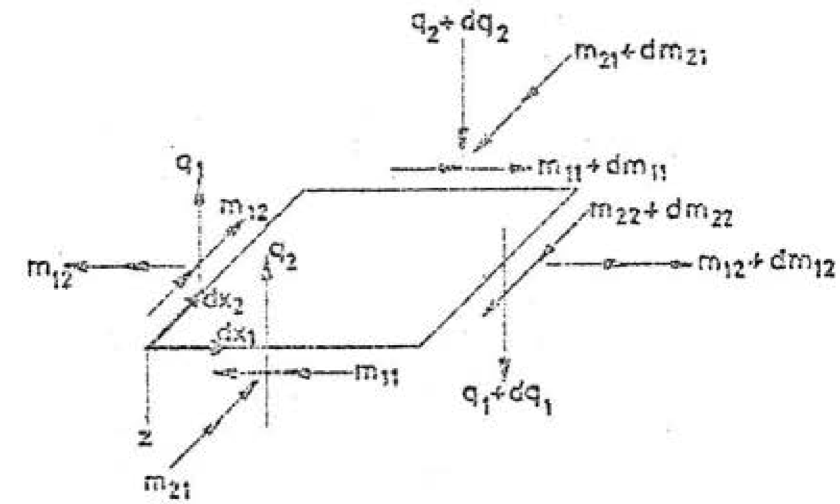


FIGURE 2._STRESS-RESULTANTS

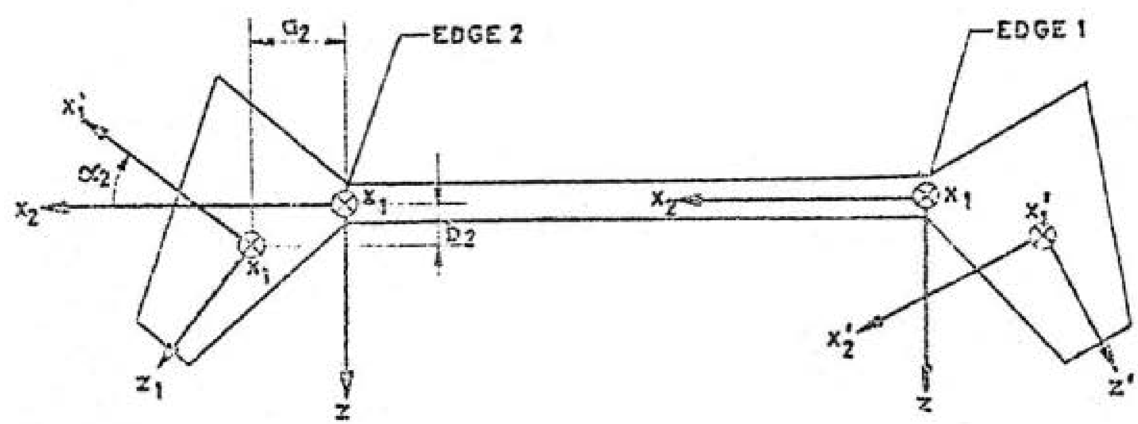
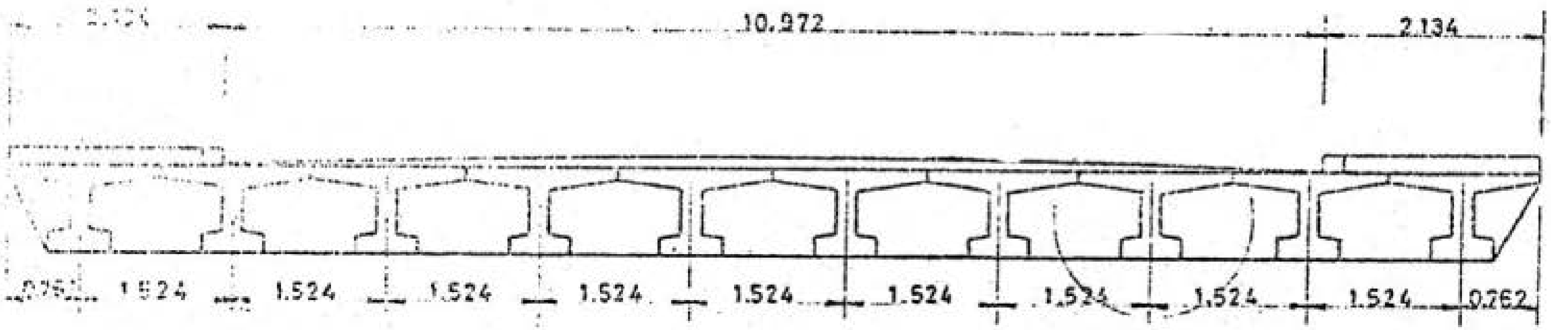
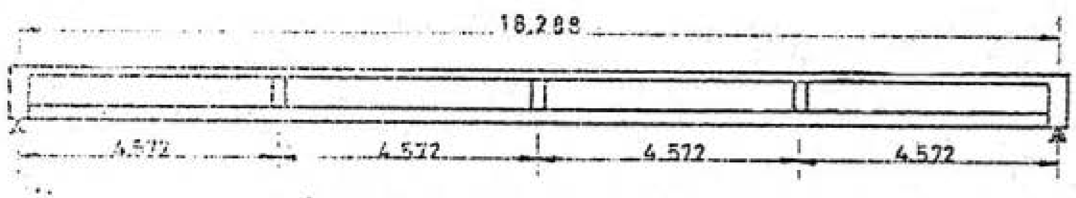


FIGURE 3.-LOCAL AXIS OF THE EDGE BEAMS



a) TRANVERSAL SECTION



b) LONGITUDINAL SECTION

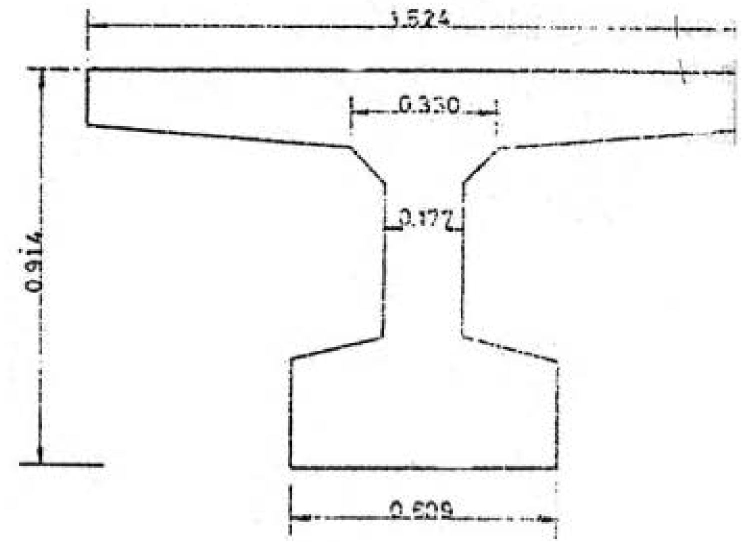


FIGURE 4.- DEFINITION OF THE BRIDGE DECK

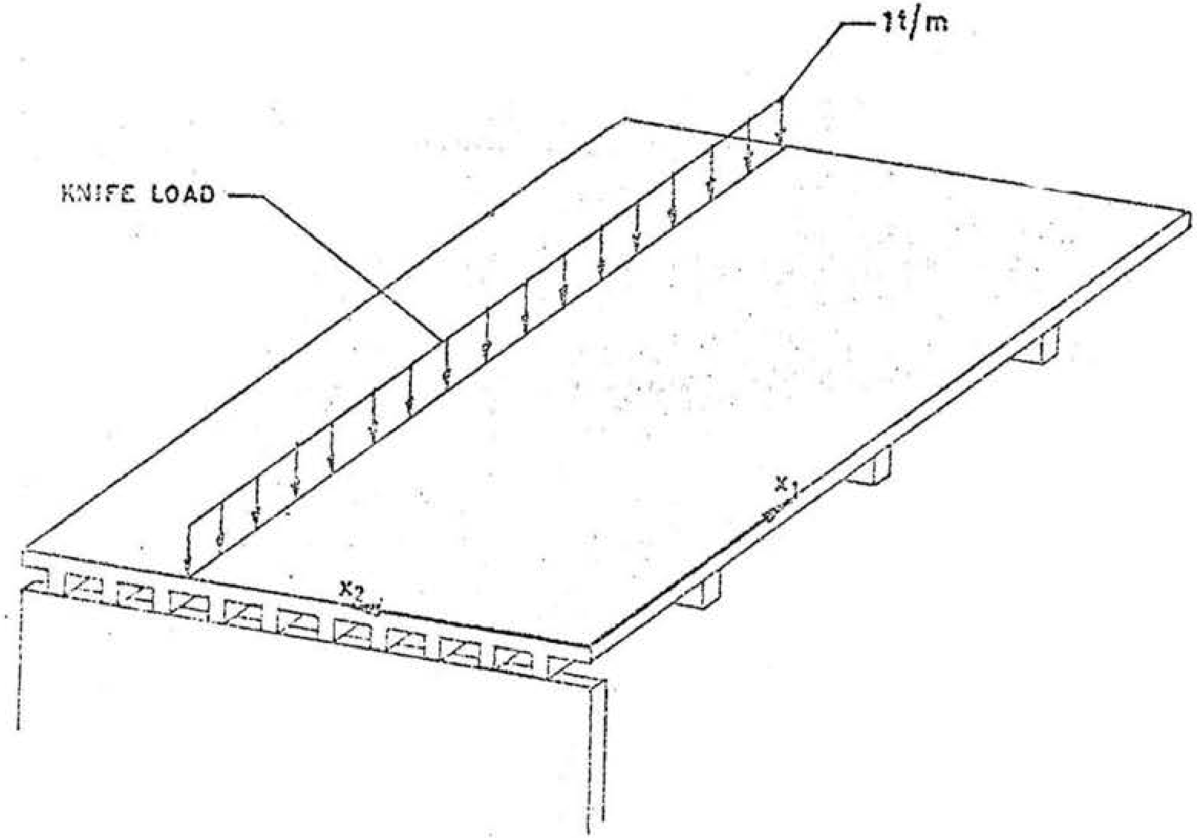


FIGURE 5_DESCRIPTION OF THE "KNIFE" LOAD

EXCENTRICITY COEFFICIENT

DEFLECTION AT 0.5 L_i^*

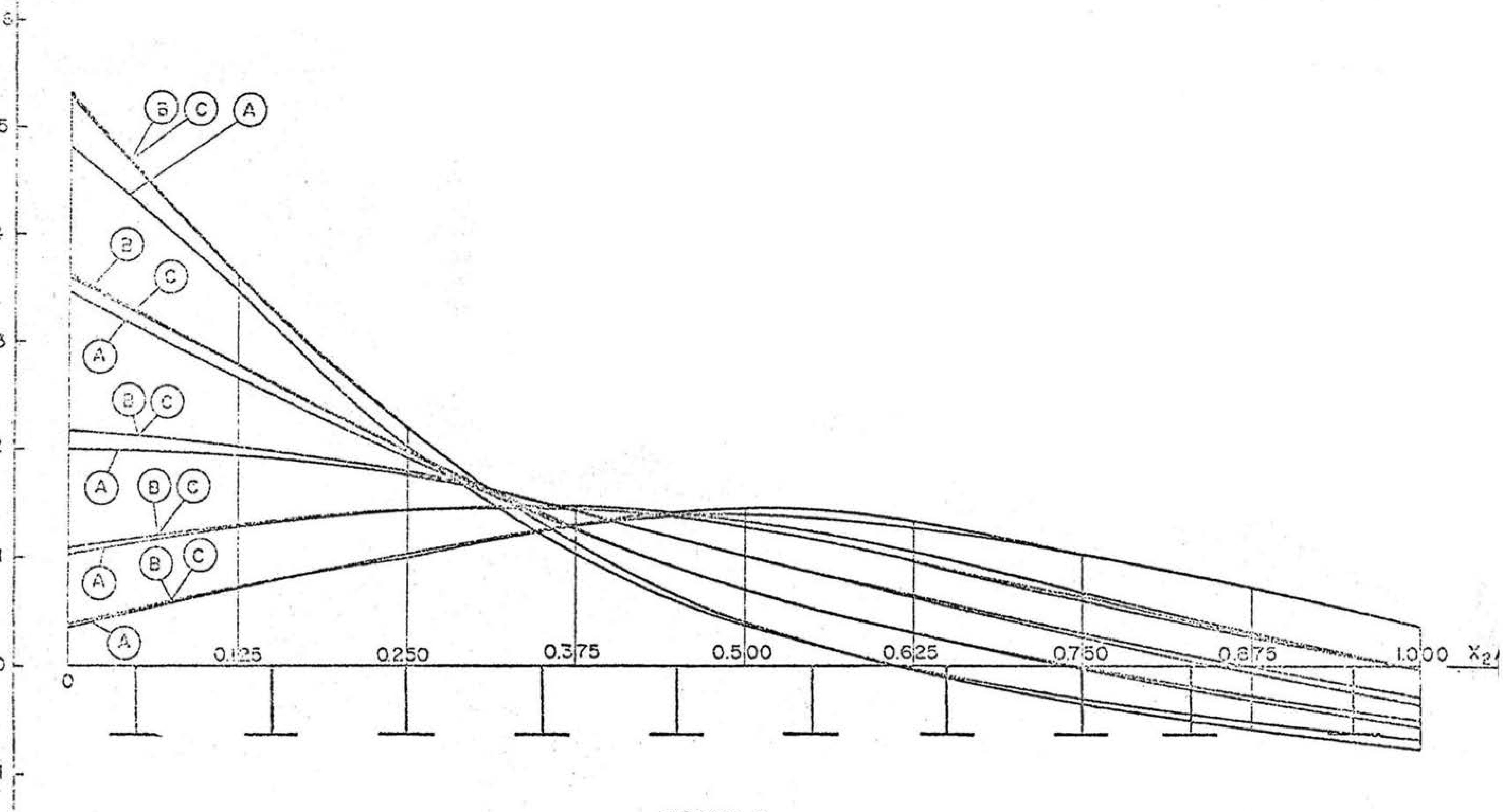


FIGURE 6

EXCENTRICITY COEFFICIENT
BENDING MOMENTS

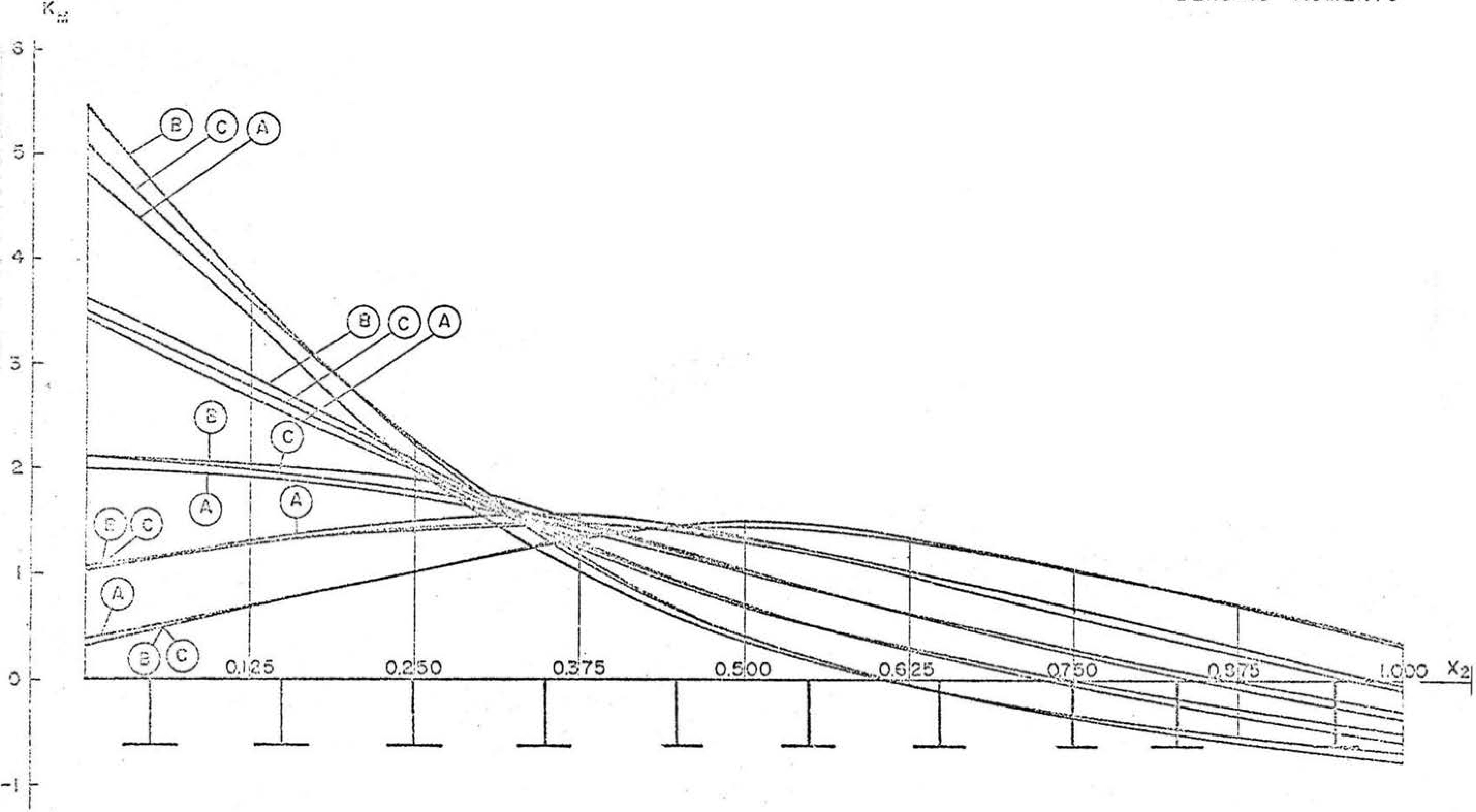


FIGURE 7

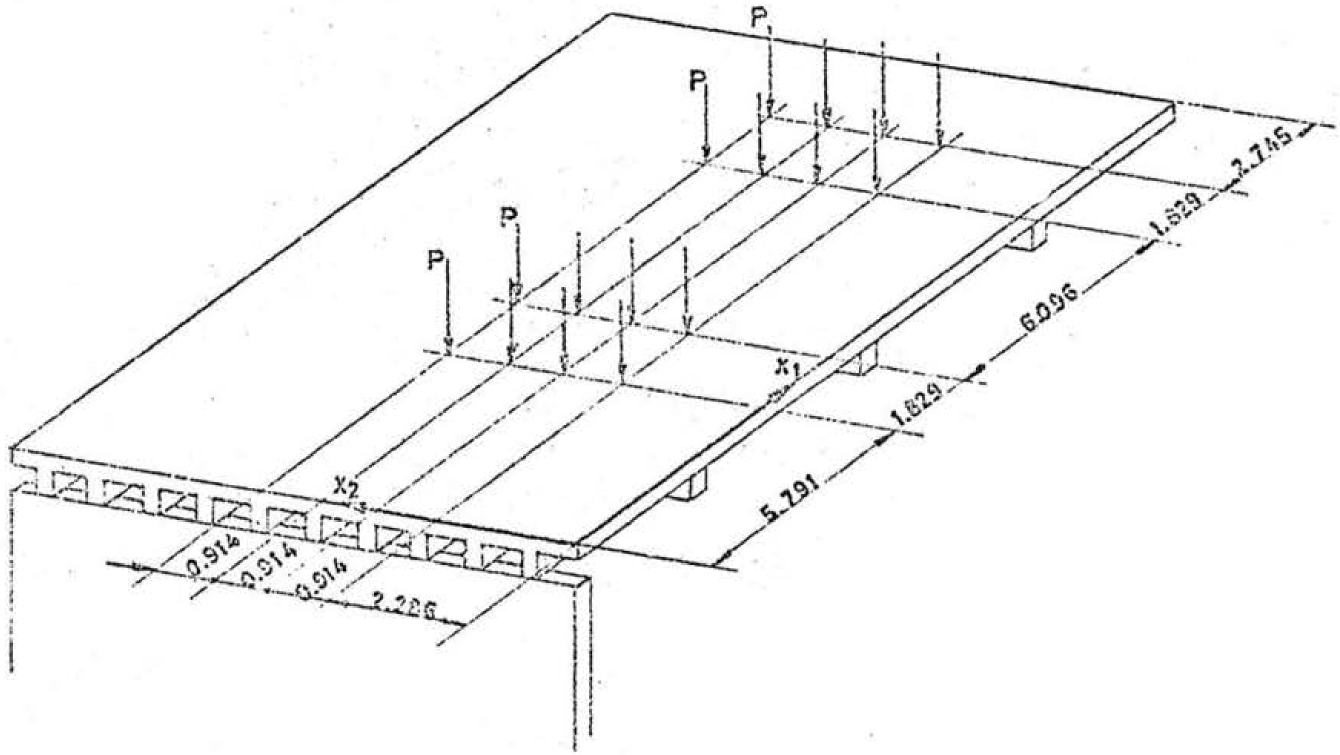


FIGURE 8.—POSITION OF THE LIVE LOAD FOR THE COMPUTATION OF m_{11}

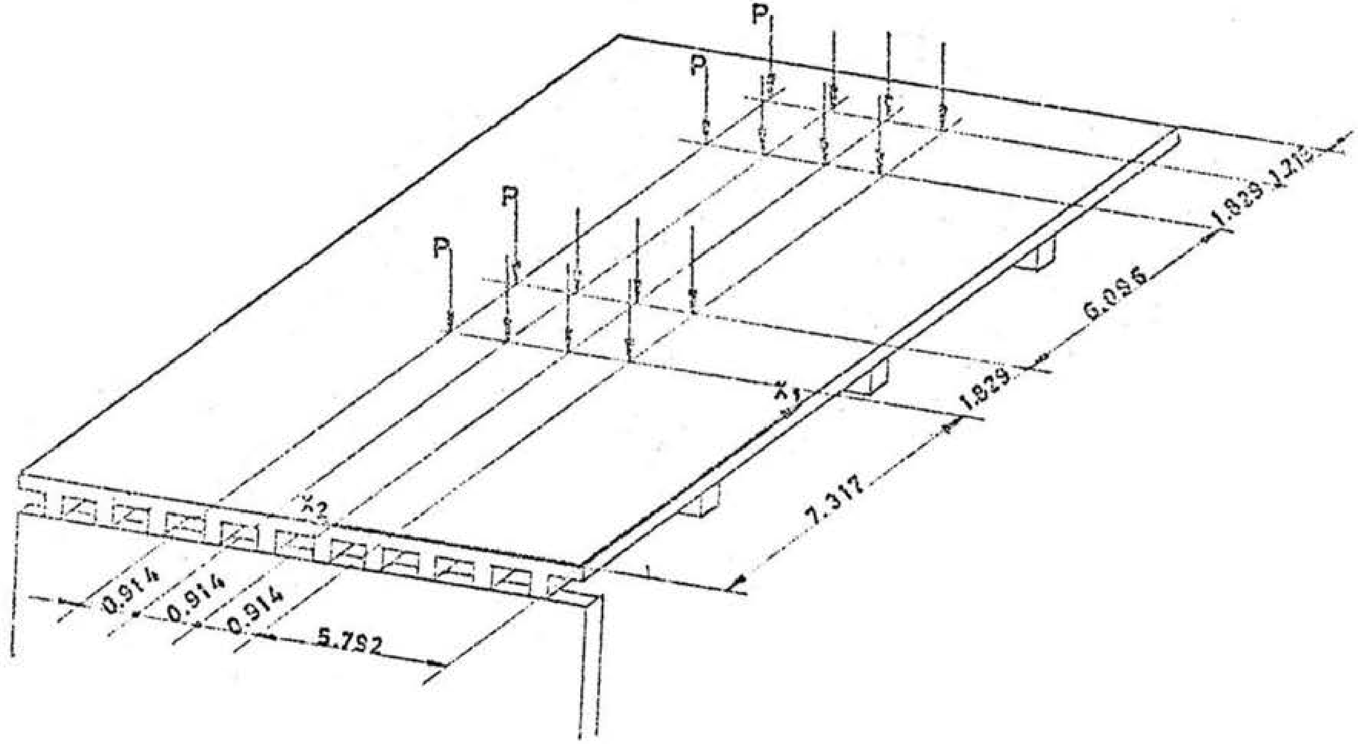
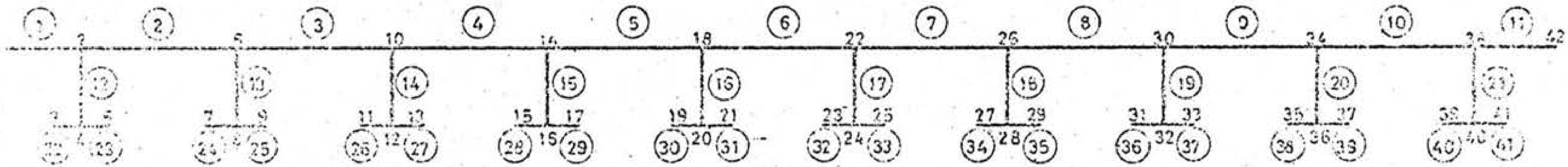


FIGURE 9.- POSITION OF THE LIVE LOAD FOR THE COMPUTATION OF m_{22}



1 - EDGE NO. 1

(1) - PLATE NO. 1

FIGURE 10.- IDEALIZATION OF THE BRIDGE DECK USING A FOLDED PLATE STRUCTURE

ORTHOTROPIC PLATE
VERTICAL DEFLECTIONS W

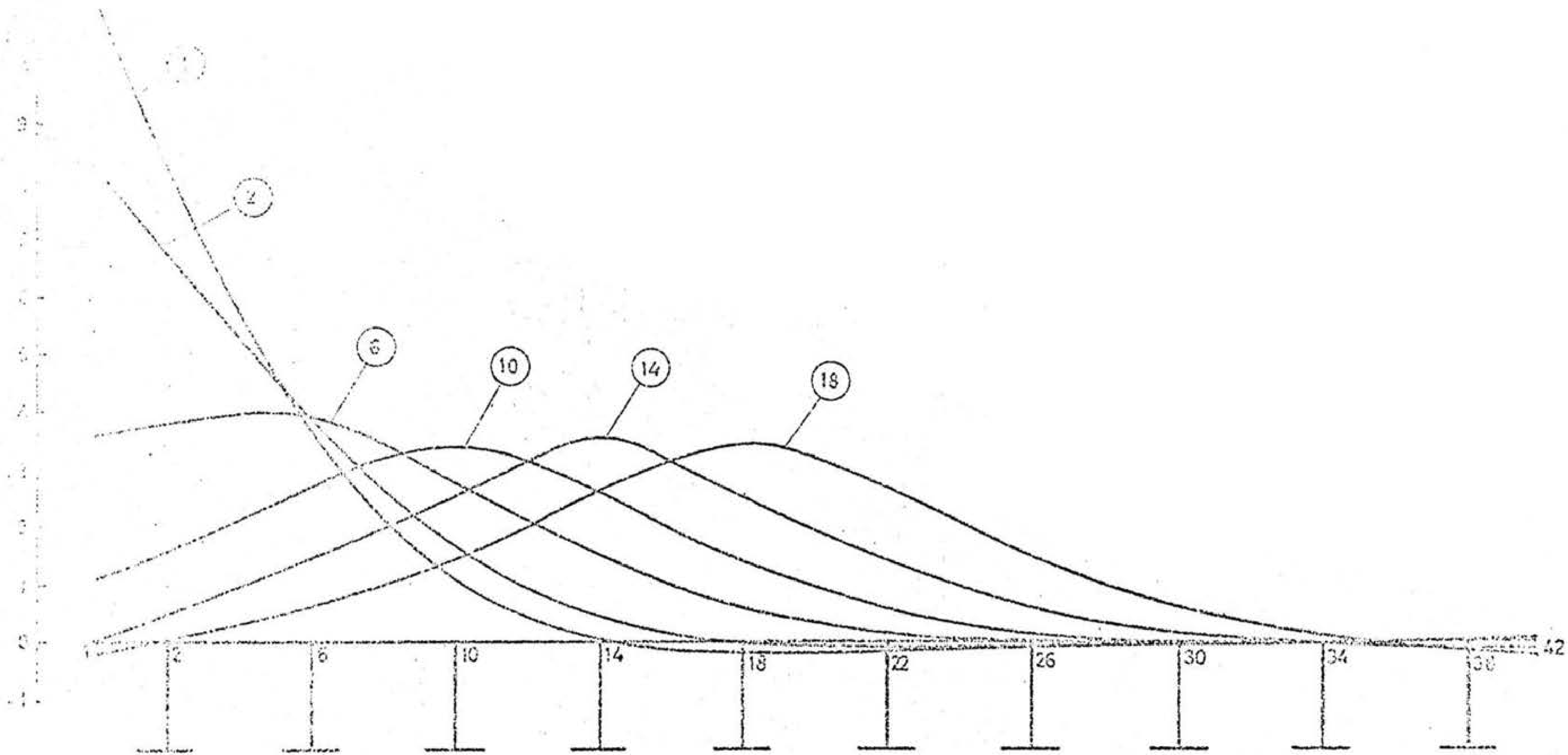


FIGURE 11

ORTHOTROPIC PLATE
BENDING MOMENTS M

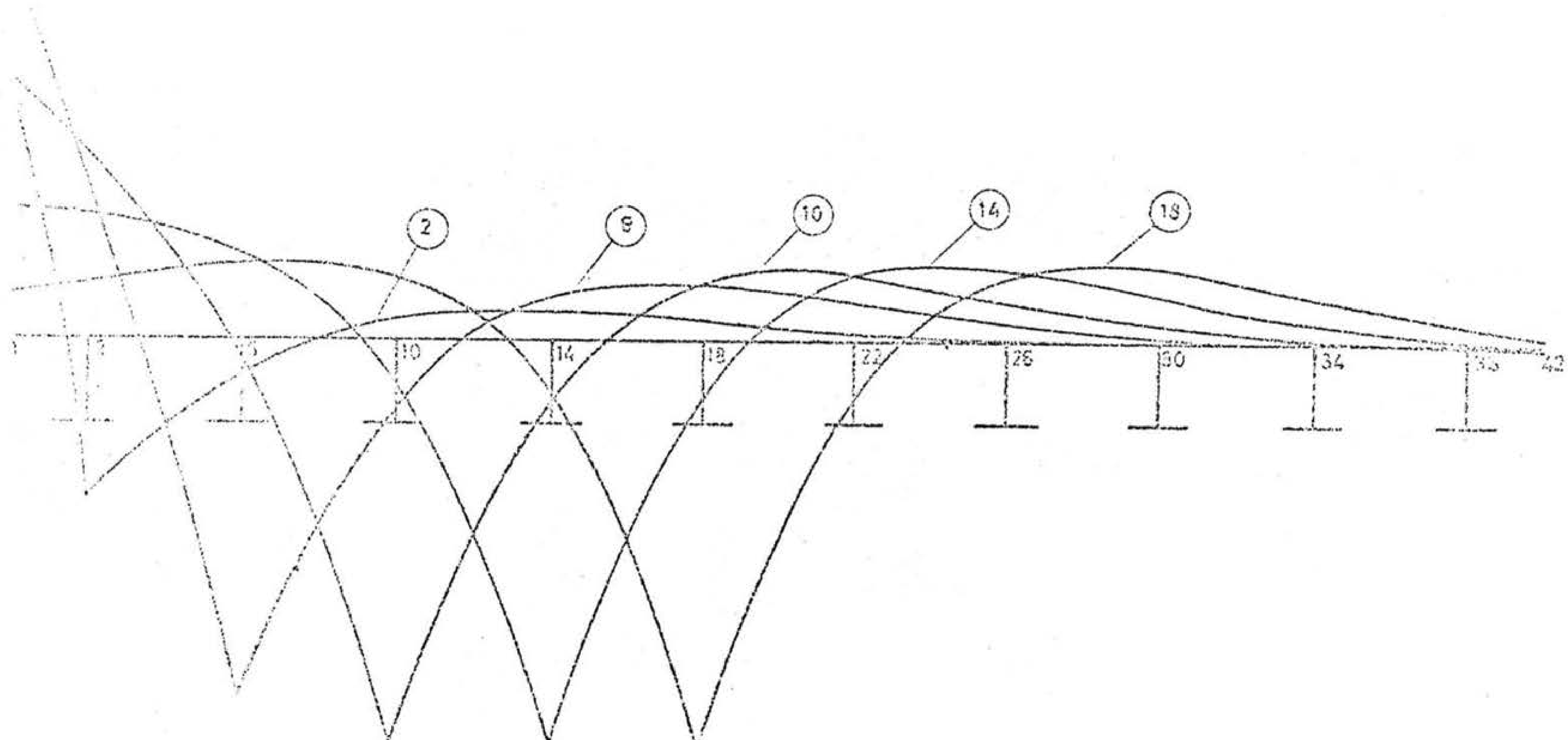


FIGURE 12

ORTHOTROPIC PLATE
LONG. BENDING STRESSES S

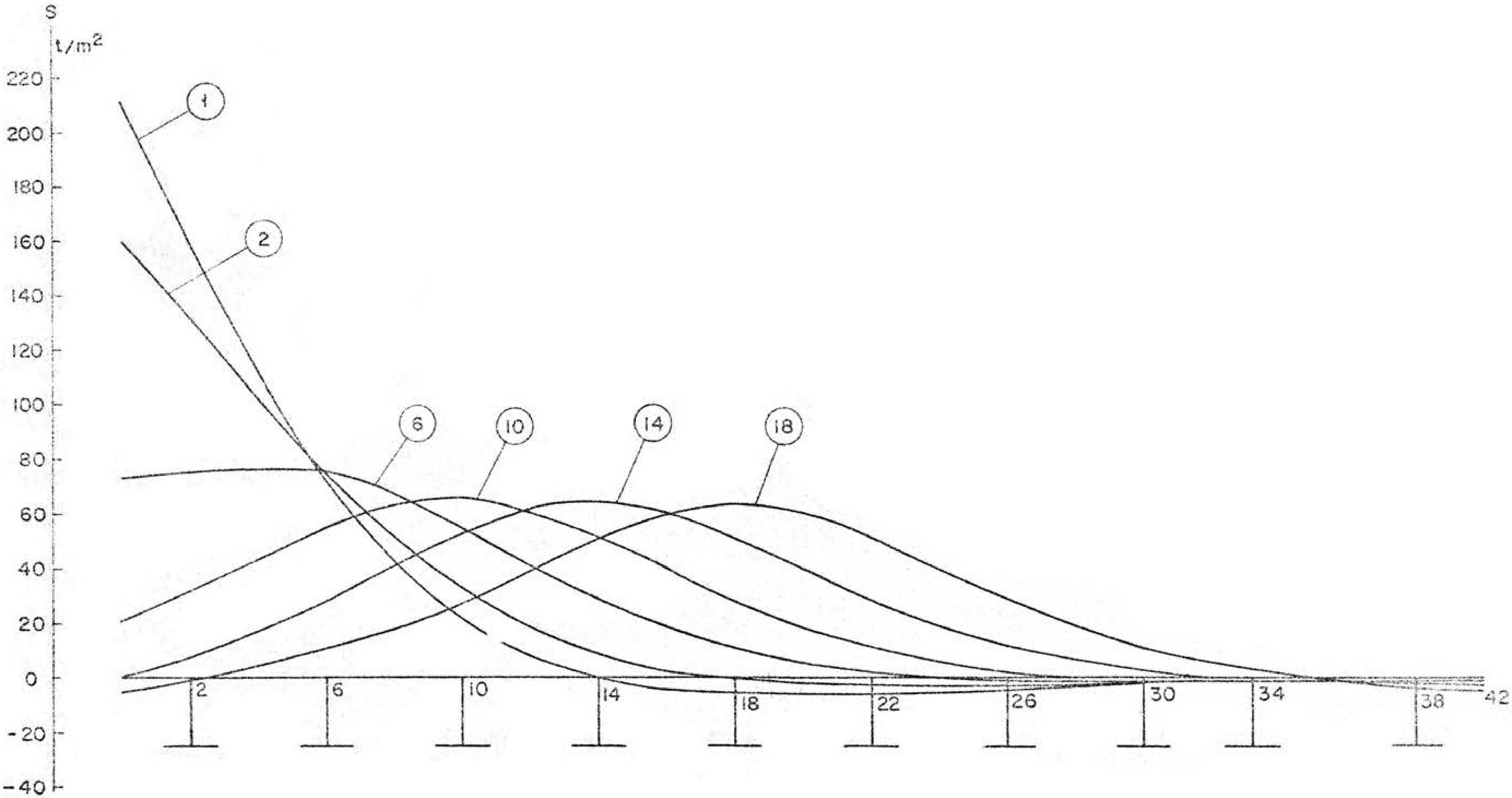


FIG IRE 13

FOLDED PLATE
VERTICAL DEFLECTIONS W

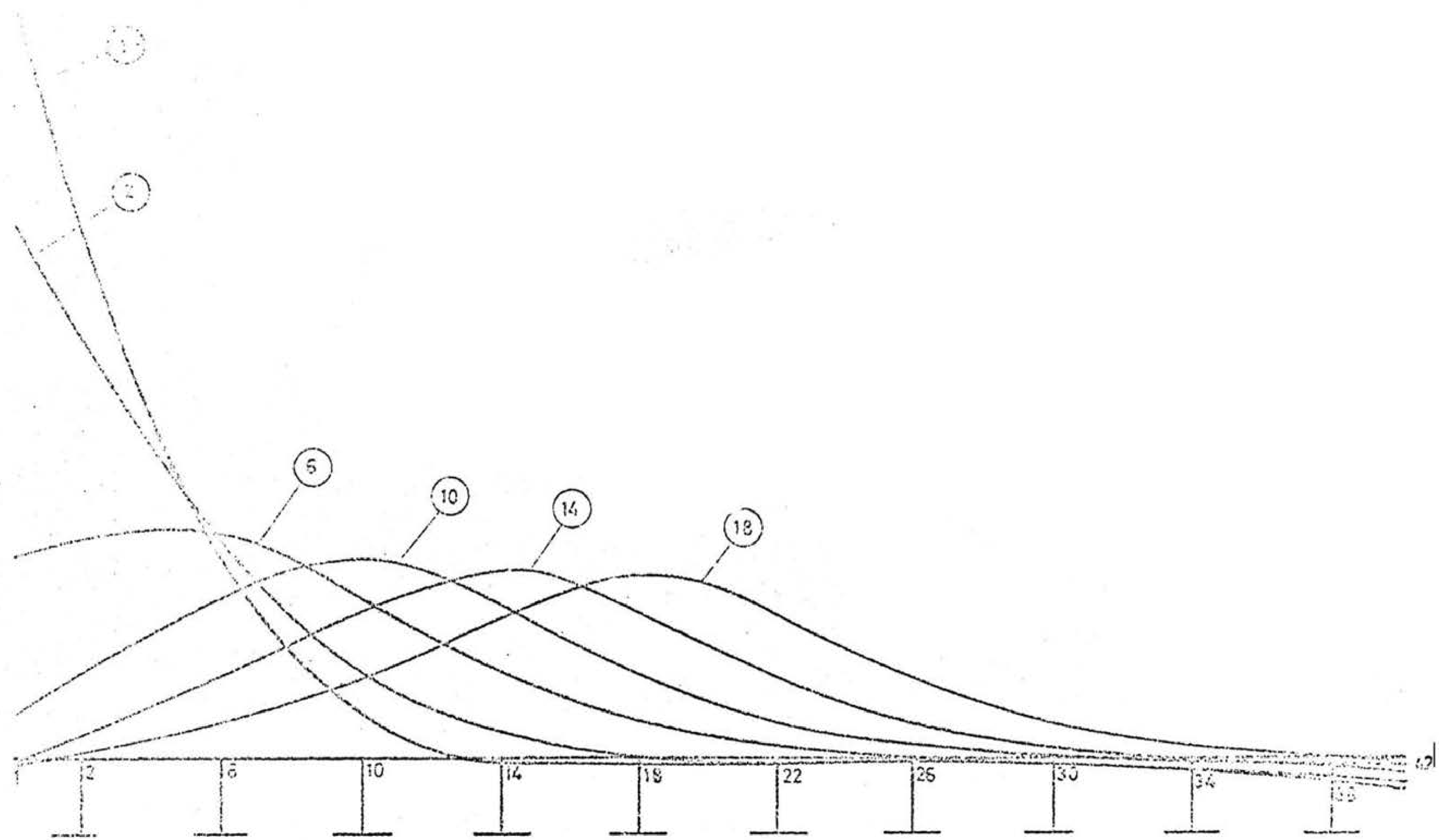


FIGURE 14

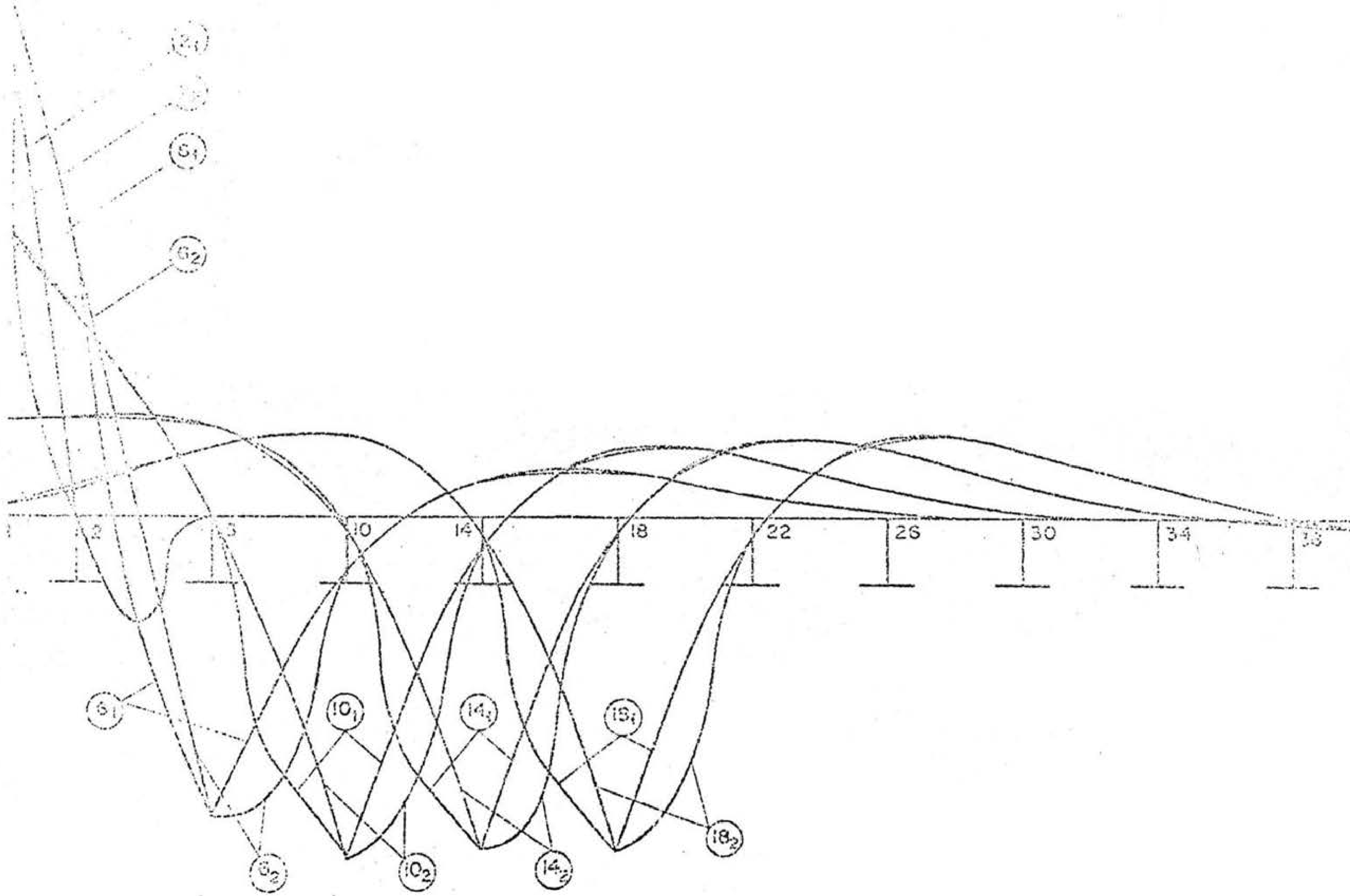


FIGURE 15

FOLDED PLATE
LONG. BENDING STRESSES

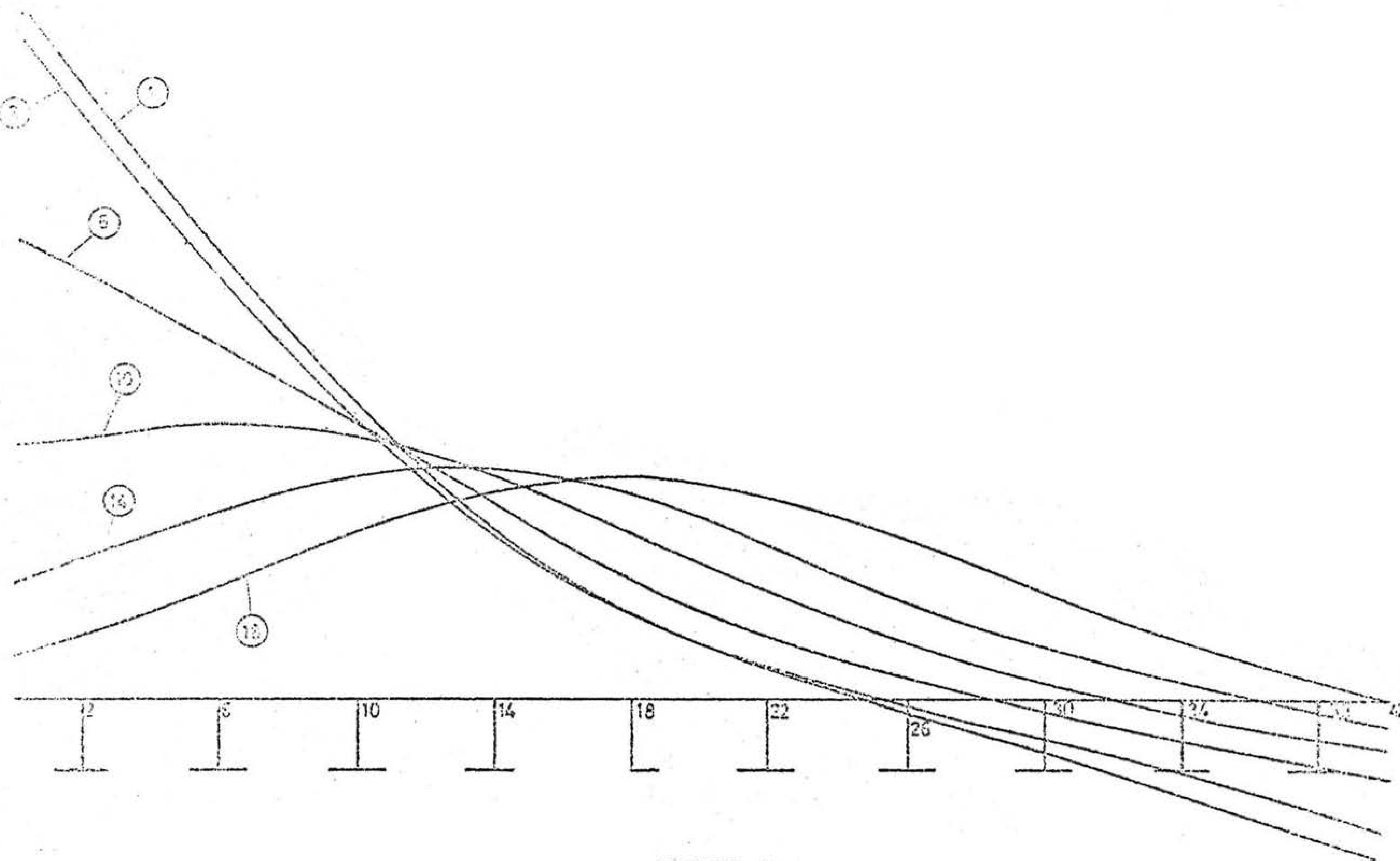


FIGURE 16

FOLDED PLATE
SHEAR T

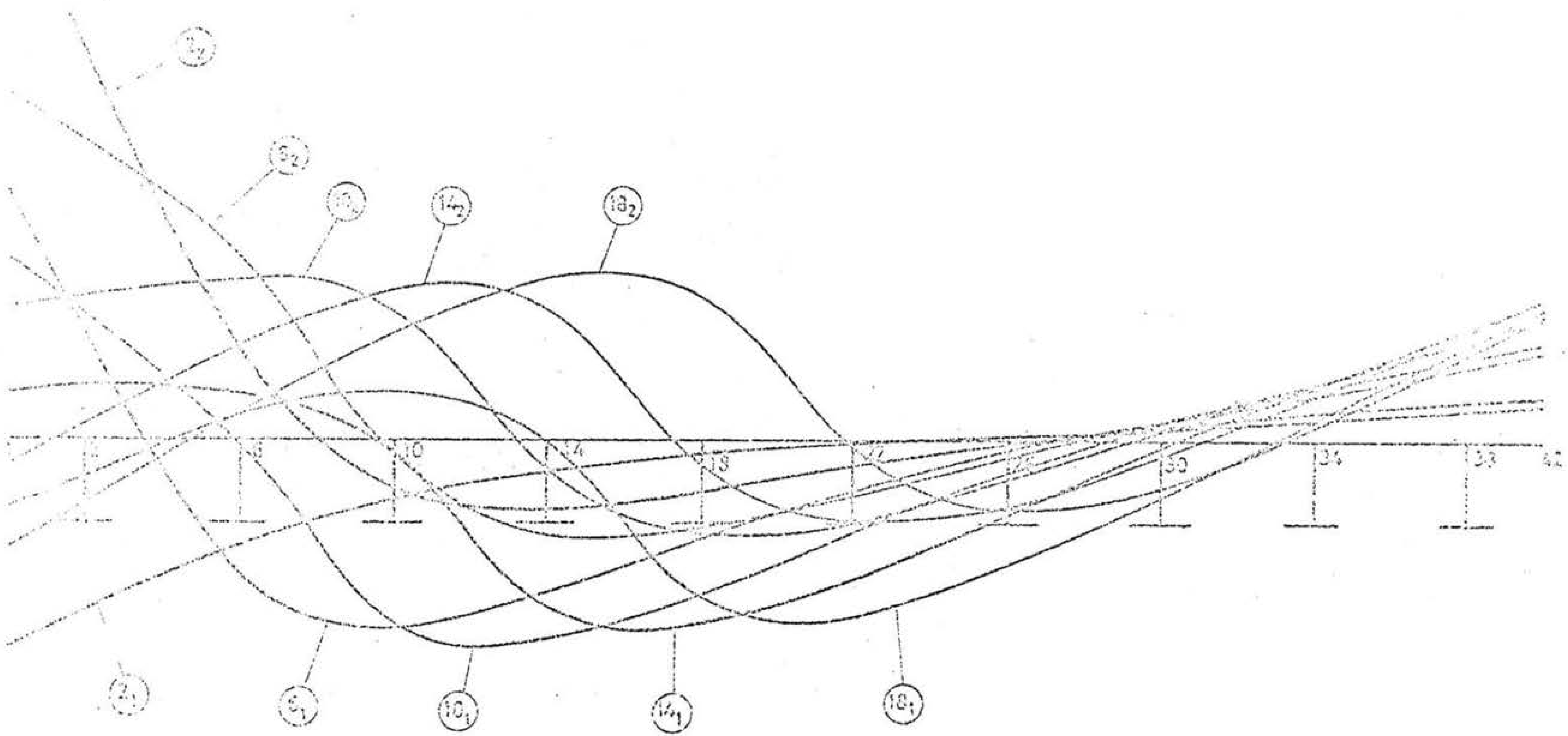


FIGURE 17